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DIFFERENTIAL EQUATIONS

ORGANIZERS: M. RUZHANSKY, S. TIKHONOV, J. WIRTH

On the solvability of the tracking problem with nonlinearly distributed control for the oscillation process

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In this talk we have investigated the tracking problem, where it is required to minimize the integral functional

$$J[u(t)] = \int_0^T \int_Q [V(t, x) - \xi(t, x)]^2 dx dt + 2\beta \int_0^T M[t, u(t)] dt, \quad \beta > 0,$$

on the set of solutions of the boundary value problem

$$\begin{aligned} V_{tt} - AV &= \lambda \int_0^t K(t, \tau) V(\tau, x) d\tau + g[t, x, u(t, x)], \quad x \in Q \subset R^n, \quad 0 < t \leq T, \\ V(0, x) &= \psi_1(x), \quad V_t(0, x) = \psi_2(x), \quad x \in Q, \\ \Gamma V(t, x) &\equiv \sum_{i,j=1}^n a_{i,j}(x) V_{x_j} \cos(\delta, x_i) + a(x) V(t, x) = 0, \quad x \in Q, \quad 0 < t \leq T. \end{aligned}$$

Here $V(t, x) \in H(Q_T)$ is a function of state of the control process; $u(t, x) \in H(Q_T)$ is a function of the control; A is an elliptic operator; $H(Y)$ is a Hilbert space of functions defined on the set Y .

The questions of unique solvability of the tracking control problem with distributed optimal control for the oscillation process described by the Fredholm integral-differential equation in partial derivatives was investigated. The uniquely solution of this problem in the form of the triple $u^0(t)$, $V^0(t, x)$, $J[u^0(t)]$ was founded. Here $u^0(t)$ is the optimal control, $V^0(t, x)$ is the optimal process, $J[u^0(t)]$ is the minimum value of the functional.

[1] Akylbek Kerimbekov, Elmira Abdyl daeva, *Optimal Distributed Control for the processes of Oscillations described by Fredholm Integro-Differential Equations*, Eurasian Mathematical Journal **V.6, No 2** (2015), 18-40.

On the nonlinear Dirac equation with an electromagnetic potential

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In a joint work with M.Okamoto (Shinshu University, Nagano) we prove smoothing and Strichartz estimates for a Dirac equation perturbed by a large potential of critical decay and regularity. In the endpoint case, we prove suitable replacements of these estimates for data of additional angular regularity. As an application we deduce global well posedness and scattering for small data in the energy space with radial symmetry, or with additional angular regularity. Moreover, for a restricted class of potentials, we can extend our results to more general large data under the sole assumption of smallness of the Lochak-Majorana chiral invariants.

[4] L. Gr. Ixaru, G. Vanden Berghe , *Exponential Fitting*, Kluwer Academic Publishers, Dordrecht, 2004.

The Cauchy problem for the generalized Zakharov-Kuznetsov equation in modulation spaces

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In this talk, we consider the well-posedness for the generalized Zakharov-Kuznetsov equation $\partial_t u + \partial_{x_1} \Delta u = \partial_{x_1} (u^{m+1})$ in modulation spaces $M_{p,q}^s(\mathbb{R}^n)$ with $n \geq 2$. In order to obtain the well-posedness, some linear estimates play crucial roles. In this talk, we mainly focus on the maximal function estimate and re-establish the estimate in the frame of modulation spaces. Then, as an application, we obtain the well-posedness in $M_{2,1}^0(\mathbb{R}^n)$ for $m \geq 4$, which is the result in a new class of functions which is not treated by that in the scaling critical Sobolev spaces.

A new conditions for the solvability of the nonlinear Fredholm integral equation

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In this paper we investigate questions of the unique solvability of the nonlinear Fredholm integral equation

$$\varphi(x) = \lambda \int_a^b K[x, t, \varphi(t)] dt. \quad (11)$$

It is established that if the following conditions are satisfied

- (1) λ is not a characteristic number of the kernel $K_\psi[x, t, \psi_i(t, \lambda)]$;
- (2) $\int_a^b K_\psi[x, t, \psi_i(t, \lambda)] \int_a^b K[t, s, f(s)] ds dt \equiv 0, \quad x \in [a, b],$
 $i = 1, 2, \dots, q.$

Then equation (1) is unique solvable. An algorithm of determining the unknown function $\psi_i(t, \lambda)$ was developed. Examples which sustain the theoretical conclusions were given.

[1] Akylbek Kerimbekov, *About the one method of solving nonlinear Fredholm integral equations*, Abstract. V Congress of the Turkic World mathematicians, Kyrgyzstan, Issyk-Kul Aurora, 5-7 June, 2014 (2014), 121.

Singular integrals with product Cauchy kernels

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One of the purposes of our talk is to present necessary and sufficient conditions both for curves and weights ensuring the boundedness of maximal double Cauchy singular integral operator in weighted Lebesgue spaces. Almost everywhere existence of double Cauchy singular integrals will be discussed.

special cases. Moreover, new Sobolev type spaces are defined on stratified Lie groups and proved an embedding theorems for these functional spaces.

- [1] N. Badiale and G. Tarantello, *A Sobolev-Hardy inequality with applications to a nonlinear elliptic equation arising in astrophysics*, Arch. Ration. Mech. Anal., **163** (2002), 259–293.
- [2] D. G. Costa., *Some new and short proofs for a class of Caffarelli-Kohn-Nirenberg type inequalities*, J. Math. Anal. Appl. **337** (2008), 311-317.
- [3] D. G. Costa. *On Hardy-Rellich type inequalities in \mathbb{R}^N* . Appl. Math. Letters, **22** (2009), 902–905.
- [4] L. D’Ambrosio. *Some Hardy inequalities on the Heisenberg group*, Differential Equations, **40**, (2004), 552–564.
- [5] Y. Di, L. Jiang, S. Shen, and Y. Jin. *A note on a class of Hardy-Rellich type inequalities*. J. Inequal. Appl., **84**, (2013), 1–6.
- [6] V. Fischer and M. Ruzhansky. *Quantization on nilpotent Lie groups*, Birkhäuser, **314**, (2016) *Progress in Mathematics*.
- [7] M. Ruzhansky and D. Suragan. *Layer potentials, Kac’s problem, and refined Hardy inequality on homogeneous Carnot groups*. arXiv:1512.02547, (2015).
- [8] M. Ruzhansky and D. Suragan. *On horizontal Hardy, Rellich and Caffarelli-Kohn-Nirenberg inequalities on stratified groups*. J. Differential Equations, **262** (2017), 1799–1821.
- [9] M. Ruzhansky and D. Suragan. *Anisotropic L_2 -weighted Hardy and L_2 -Caffarelli-Kohn-Nirenberg inequalities*. Commun. Contemp. Math., to appear, <https://arxiv.org/abs/1610.07032>.
- [10] M. Ruzhansky and D. Suragan. *Local Hardy and Rellich inequalities for sums of squares of vector fields*. Adv. Diff. Equations, to appear, <https://arxiv.org/abs/1601.06157>.

On the solvability of the tracking problem under nonlinear boundary control of the thermal processes described by Volterra integro-differential equations

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In this talk we have investigated of the tracking problem, where it is required to minimize the quadratic functional

$$J[u(t)] = \int_0^T \int_0^1 [\nu(t, x) - \xi(t, x)]^2 dx dt + \beta \int_0^T u^2(t) dt, \quad \beta > 0, \quad (12)$$

on the set of solutions of the boundary value problem

$$\nu_t = \nu_{xx} + \lambda \int_0^t K(t, \tau) \nu(\tau, x) d\tau \quad 0 < x < 1, \quad 0 < t \leq T, \quad (13)$$

$$\nu(0, x) = \psi(x), \quad 0 < x < 1, \quad (14)$$

$$\nu_x(t, 0) = 0, \quad \nu_x(t, 1) + \alpha \nu(t, 1) = p[t, u(t)], \quad 0 < t \leq T. \quad (15)$$

Here $\nu(t, x) \in H(Q)$ is a function of state of the control process; $u(t) \in H(0, T)$ is a function of the control; $H(Y)$ is a Hilbert space of functions defined on the set Y .

An algorithm for constructing solutions tracking problem of the nonlinear optimization for the thermal process described by the Volterra integro-differential equation in partial derivatives was developed.

- [1] Akylbek Kerimbekov, Elmira Abdylbaeva, *On the Solvability of a Nonlinear Tracking Problem under Boundary Control for the Elastic Oscillations described by Fredholm Integro-Differential Equations*, System Modeling and Optimization [27th IFIP TC 7 Conference, CSMO 2015 Sophia Antipolis, France, June 29 - July 3, 2015 Revised Selected Papers] (2017), 312-321.